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Dynamic scaling for a competitive growth process: random deposition versus ballistic deposition

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Abstract

A random-ballistic deposition model where particles are aggregated according to the rules of ballistic deposition with probability p and following a random deposition process with probability $1 - p$, respectively, is proposed and studied. Based on extensive numerical simulations a dynamic scaling ansatz for the interface width $W(L, t, p)$ as a function of lattice side L, time t and p is formulated. Three new exponents, which can be linked to the standard growth exponent of ballistic deposition by means of a new scaling relation, are identified.

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1. Introduction

The study of the morphology, structure and other physical and chemical properties of growing interfaces has been a subject of intensive research in recent years [1–4]. This interest is mainly due to the fact that evolving interfaces can be found in a great variety of physical, chemical and biological systems and processes. For example, film growth either by vapour deposition, chemical deposition or molecular-beam epitaxy [1, 5], bacterial growth [6], propagation of reaction fronts in catalysed reactions [7], propagation of forest fires [8] and diffusion fronts [9], should be mentioned.

In spite of the great progress achieved in the understanding of interface behaviour, e.g. due to the introduction of phenomenological dynamic scaling [10], the nonlinear Kardar–Parisi–Zhang (KPZ) equation [11], renormalization group methods [1, 12], etc, there are still many controversies and open questions [13]. Within this context it should be recognized that considerable effort has been devoted to the study of growth models involving one kind of particle [1–4]. In contrast, less attention has been drawn to the study of the dynamic of competitive processes. However, these processes are significant to the growth of real materials in at least two different ways: (a) when the growing process involves two or more kinds of particles and (b) when deposition of a single kind of particle is considered, but such a type of particle may undergo different growing mechanisms.

In fact, the construction of materials with specific electronic, mechanical, optical or magnetic properties, often requires the deposition of more than one kind of atoms or molecules. Thus, one example of case (a) arises from the deposition of alloys or systems with impurities (see, e.g., [14–19] and references therein). In this case, there may be different interactions between different kinds of particles and the growing mechanisms change [14–19]. Based on these ideas, Cerdeira *et al* [14–17] have studied various models for binary systems involving competitive random-like (RLD) and ballistic-like (BLD) deposition. In these models the deposition of two kinds of particles A and C (particle A with probability P and particle C with probability $1 - P$) on a d-dimensional substrate have been studied. The models exhibit interesting crossover behaviour between both extreme model cases when P is varied (see [14–17] for additional details). Recently, the scaling behaviour of a two-component surface-growth model has been studied by Kotrla *et al* [19]. This study addresses the relationship between kinetic roughening and phase ordering in a (1 + 1)-dimensional single-step solid-on-solid model with Ising-like interactions between two components. An interesting finding is that phase ordering leads to faster kinetic roughening than in the homogeneous case. However, this behaviour is time dependent and crosses over to the ordinary behaviour characteristic of homogeneous samples [19]. Very recently, the interplay between surface roughening and phase separation during the growth of a binary mixture in (1 + 1)-dimensions has also been studied by means of numerical simulations and field-theoretical calculations [18].

On the other hand, considering deposition of one kind of particle (case b), Pellegrini *et al* [20, 21] have studied a ballistic model of surface growth that considers ‘sticky’ and ‘sliding’ particles. The model interpolates between a standard ballistic model when only sticky particles are deposited (with probability $P = 1$) and a completely restructured ballistic model for $P = 0$ when only unrestricted sliding particles are allowed to become attached to the sample. Using this model Pellegrini *et al* [20, 21] have given evidence of a roughening transition in dimensions $d = 3$ and 4, while such a kind of transition is no longer observed in $d = 2$. Also, this surface roughening transition is accompanied by a transition in the bulk of the sample that is characterized by a singularity in the compactness of the aggregate [20, 21].

Another scenario for the deposition of a single kind of particle in a competitive process arises from the growth of polycrystalline films [5]. In this case, particles deposited in the central area of small crystals undergo restricted diffusion, while particles deposited close to the intercrystalline gaps can experience intercrystalline diffusion and consequently different adsorption mechanisms have to be considered [5].

It should also be mentioned that in a related context of competitive growing processes, Derrida and Dickman [22] have studied the interface formed by the competitive growth of different Eden clusters. Also, one of us has recently investigated the properties of the interface generated by the collision of two (Eden) growing interfaces [24].

The aim of this paper is to explore an alternative approach to the study of competitive dynamic processes. In fact, we propose a model in which two different growth dynamics, namely random deposition and ballistic deposition, undergo a stochastic competitive process. It is worth mentioning that this process can be well described by a generalized dynamic phenomenological scaling approach that requires an additional set of new exponents.

The manuscript is organized as follows. Firstly, in section 2, the model is described and the standard dynamic scaling approach briefly reviewed. Subsequently, in section 3 our numerical results are presented and discussed. A generalized scaling approach is developed during the discussion of the results. Finally, our conclusions are stated in section 4.

2. Description of the model, the simulation method and the standard dynamic scaling approach

We studied a discrete growth model, namely the BD/RD model, where particles are aggregated according to the rules of ballistic deposition (BD) with probability p and according to the rules of random deposition (RD) with probability $(1 - p)$. Simulations were performed in (1 + 1)-dimensions using lattices of side L and assuming periodic boundary conditions.

RD is probably the simplest growth model: from a randomly chosen site over the surface of length L, a particle falls vertically until it reaches the top of the column under it, whereupon it is deposited. BD was previously introduced as a model of colloidal aggregates [1]. The lattice version of BD is rather simple to describe: a particle is released from a random position above the surface of length L. Of course, such a particle is initially located at a distance larger than the maximum height of the interface. Subsequently, the particle follows a straight vertical trajectory until it reaches the surface, whereupon it sticks. Snapshot configurations of RD and BD aggregates, and further details on the deposition rules can be found in [1].

In order to study the growth process quantitatively it is convenient to introduce some definitions. The interface of the aggregate is defined as the set of particles that are placed at the highest position of each column. So, the mean height of the interface, $\langle h(t)\rangle$, at time t is given by

$$\langle h(t)\rangle \equiv \frac{1}{L}\sum_{i=1}^{L} h(i,t) \tag{1}$$

where $h(i,t)$ is the height of the ith column at time t. The interface width, $W(L,t)$, which characterizes the roughness of the interface, is also defined by the rms fluctuation in height,

$$W(L,t) \equiv \sqrt{\frac{1}{L}\sum_{i=1}^{L}[h(i,t) - \langle h(t)\rangle]^2}. \tag{2}$$

The Family–Vicsek scaling relation [10] has proved to be very successful for the description of the dynamic evolution of a growing interface, namely

$$W(L,t) \propto L^{\alpha} f\left(\frac{t}{L^Z}\right) \tag{3}$$

where the exponents α and Z are called the roughness exponent and the dynamic exponent, respectively. Also, $f(u)$ is a suitable scaling function that behaves as follows: (a) $f(u) =$ constant for $u \gg 1$ or, in other words, the interface width saturates for a long enough time and (b) $f(u) \propto u^{\beta}$ for $u \ll 1$. The former condition implies that $W(t) \propto t^{\beta}$ may hold during the short-time regime, where β is the growth exponent. A scaling relationship can easily be derived so that $Z = \frac{\alpha}{\beta}$ and only two independent exponents remain.

For the RD model, $W(t)$ does not saturate due to the lack of lateral correlations, so

$$W(t) \propto t^{\beta_{RD}} \tag{4}$$

independent of L with $\beta_{RD} = \frac{1}{2}$. In contrast, the BD process causes the development of lateral correlations and, therefore,

$$W(t) \propto t^{\beta_{BD}} \qquad L \to \infty \tag{5}$$

and

$$W(t) \propto L^{\alpha_{BD}} \qquad t \to \infty \tag{6}$$

with $\beta_{BD} = \frac{1}{3}$, $\alpha_{BD} = \frac{1}{2}$, which gives $Z_{BD} = \frac{3}{2}$.

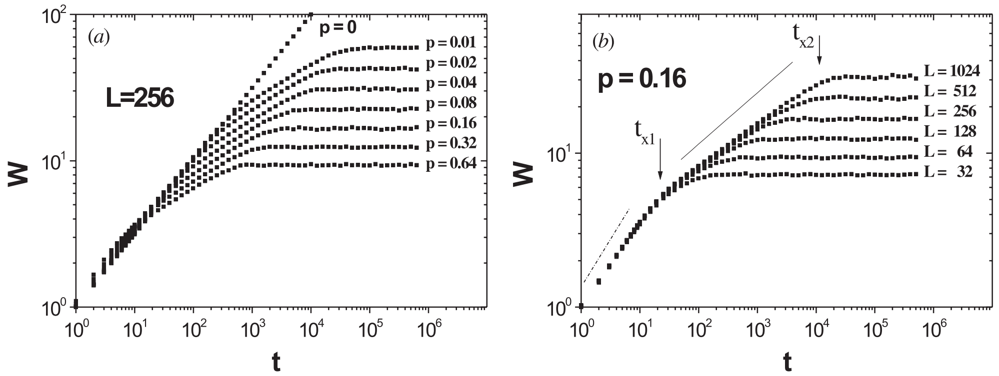

Figure 1. Log–log plots of the interface width (W) versus time for the BD/RD model as obtained for: (a) $L = 256$ and different values of p as indicated in the figure, and (b) $p = 0.16$ and lattices of different size, as indicated in the figure. In figure (b) the arrows show the location of t_{x1} and t_{x2} for the data corresponding to $L = 1024$. Also in (b) the broken (full) curve has slope $\beta_{RD} = \frac{1}{2}$ ($\beta_{BD} = \frac{1}{3}$), respectively, and have been drawn for the sake of comparison. More details in the text.

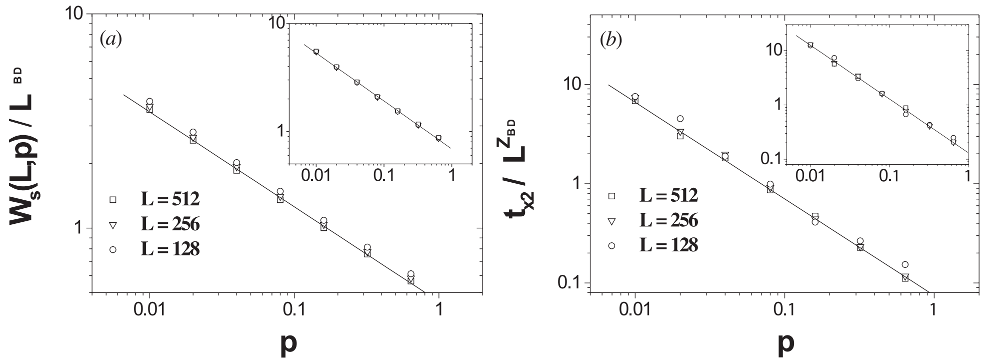

Figure 2. (a) Log–log plots of $W_S(L, p)L^{-\alpha_{BD}}$ versus p obtained for lattices of different size, as indicated in the figure, and assuming $\alpha_{BD} = \frac{1}{2}$. The full line has slope $\delta = 0.45$ and corresponds to the best fit of the data. The inset shows the same scaled plot but obtained assuming $\alpha'_{BD} = 0.43$. Again, the full line with slope $\delta = 0.45$ shows the best fit of the data. (b) Log–log plots of $t_{x2}L^{-Z_{BD}}$ versus p obtained for lattices of different size, as indicated in the figure, and assuming $Z_{BD} = \frac{3}{2}$. The full line has slope $y = 0.97$ and corresponds to the best fit of the data. The inset shows the same scaled plot but obtained assuming $Z'_{BD} = 1.4$. Again, the full line with slope $y = 0.97$ shows the best fit of the data. More details are given in the text.

3. Results and discussion

Figure 1(a) shows plots of W versus t obtained for the BD/RD model using different values of p. For $p = 0$ the monotonic growth of W characteristic of the RD process can be observed, while introducing the ballistic competition, saturation occurs. However, it is worth mentioning that the saturation value of W depends sensitively on p. Figure 1(b) shows plots of W versus t obtained for lattices of different size but keeping $p = 0.16$ constant. Here, as well as in figure 1(a) three different regimes and the corresponding crossovers can easily be observed. For short times, say $t < t_{x1}$, the growth is dominated by the RD process since correlations have not already developed. So, it follows that $W(t) \propto t^{\beta_{RD}}$, $t < t_{x1}$. During an intermediate time regimen, say $t_{x1} < t < t_{x2}$, correlations have developed and the BD process

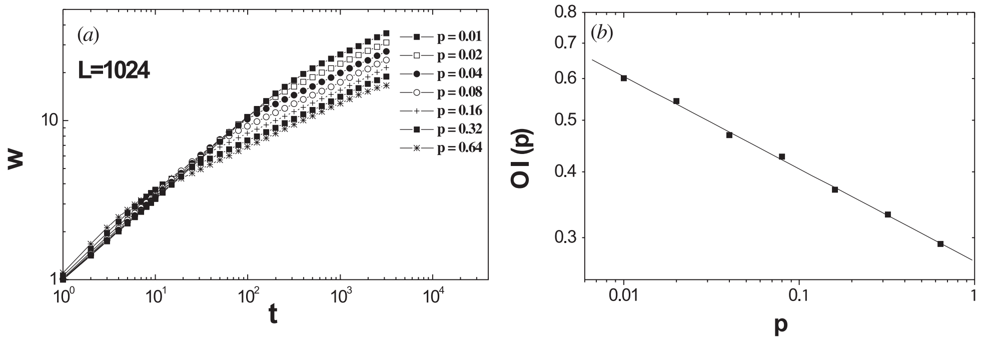

Figure 3. (*a*) Log–log plot of W versus t for the BD/RD model obtained for $L = 1024$ and different values of p as indicated in the figure. (*b*) Log–log plot of the ordinate intersection (OI) obtained from figure 3(*a*) versus p. The straight line with slope $\gamma = 0.17$ corresponds to the best fit of the data. More details in the text.

dominates. So, $W(t) \propto t^{\beta_{BD}}$ results. Finally, for $t > t_{x2}$, correlations can no longer develop due to the geometrical constraint of the lattice size and saturation is observed. In order to outline a phenomenological dynamic scaling approach, we propose the following ansatz for the saturation value of the interface width ($W_s(L, p)$) and the crossover time t_{x2} :

$$W_s(L, p) \propto L^{\alpha_{BD}} p^{-\delta} \qquad (p > 0) \tag{7}$$

and

$$t_{x2}(L, p) \propto L^{Z_{BD}} p^{-y} \qquad (p > 0) \tag{8}$$

where δ and y are exponents. Figure 2(a) shows log–log plots of $W_s(L, p)/L^{\alpha_{BD}}$ versus p. Using the exact value $\alpha_{BD} = \frac{1}{2}$, straight lines are observed, in agreement with equation (7), and the best fit gives the slope $\delta \cong 0.45 \pm 0.01$. However, a rather small systematic deviation of the data, according to the size of the lattice, is observed: the larger the lattice, the smaller the ordinate. This behaviour may be due to corrections of scaling of higher order that we have neglected in equation (7). On the other hand, using the roughness exponent obtained by fitting our data $\alpha'_{BD} = 0.43 \pm 0.05$, e.g. from figure 1($b$), data collapsing is excellent, as shown in the inset of figure 2(a). In this case, the obtained slope is also $\delta \cong 0.45 \pm 0.01$. So, both fits shown in figure 2(a) point out that equation (7) holds. Figure 2(b) also shows log–log plots of $t_{x2}/L^{Z_{BD}}$ versus p. Again, systematic deviations are observed when using the exact value of $Z_{BD} = \frac{3}{2}$ but excellent collapse is found using the value of $Z'_{BD} = 1.4 \pm 0.1$ obtained by fitting our data. Therefore, our assumption in equation (8) is also validated. The best fit of the data yields $y \cong 0.97 \pm 0.02$.

Figure 3(a) shows log–log plots of W versus t where both the short- and intermediate-time behaviour of the BD/RD model can clearly be observed. Results were obtained for different values of p keeping $L = 1024$ constant. As has already been mentioned, the short-(intermediate-) time behaviour is dominate by the RD (BD) process, respectively. The fact that the straight lines corresponding to the intermediate regime are equally spaced and have slopes $\beta_{BD} = \frac{1}{3}$ (for $t > t_{x1}$) (note that p varied in powers of 2) suggests the operation of a power-law behaviour that we assume to be of the form

$$W(t, p) \propto t^{\beta_{BD}} p^{-\gamma} \qquad (t_{x1} < t < t_{x2}) \tag{9}$$

where γ is an exponent.

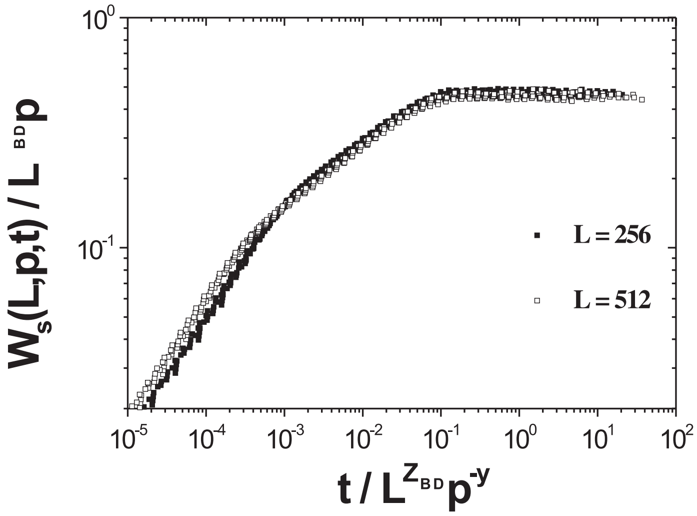

Figure 4. Log–log plot of $W(t, L, p)L^{-\alpha_{BD}} p^{\delta}$ versus $t/L^{Z_{BD}} p^{-y}$ obtained for different values of p ($0.01 \leqslant p \leqslant 0.64$) and lattices of size $L = 256$ and 512, as indicated in the figure.

Evaluating the ordinate intersection $OI(p)$ for different values of p in figure 3(a), it is possible to draw a plot of $OI(p)$ versus p as shown in figure 3(b), which yields the exponent $\gamma = 0.17 \pm 0.01$.

Our numerical results supporting the assumptions of equations (7)–(9), lead us to propose the following phenomenological dynamic scaling ansatz for the BD/RD model:

$$W(t, L, p) \propto L^{\alpha_{BD}} p^{-\delta} F\left(\frac{t}{L^{Z_{BD}} p^{-y}}\right) \qquad p > 0 \quad t > t_{x1} \quad L \to \infty \quad (10)$$

where $F(u)$ is a suitable scaling function such as: (a) $F(u) =$ constant for $u \gg 1$; in this way equation (7) is recovered for every saturated interface width; and (b) $F(u) = u^{\beta_{BD}}$ for $u \ll 1$. Therefore, equation (9) can be recovered if the following scaling relationship between the exponents holds:

$$y\beta_{BD} - \delta + \gamma = 0 \qquad (11)$$

where the identity $Z_{BD} = \alpha_{BD}/\beta_{BD}$ has been used. Using the exact value $\beta_{BD} = \frac{1}{3}$ and our estimations for y, δ and γ we obtain $y\beta_{BD} - \delta + \gamma = 0.04 \pm 0.03$. Also, using our best fit for $\beta'_{BD} \approx 0.3$ we obtain $y\beta'_{BD} - \delta + \gamma = 0.01 \pm 0.03$. These results lead us to conjecture the following exact (rational) values for the new exponents:

$$y \equiv 1 \quad (0.97 \pm 0.02) \qquad \delta \equiv \tfrac{1}{2} \quad (0.45 \pm 0.01) \qquad \gamma \equiv \tfrac{1}{6} \quad (0.17 \pm 0.01) \qquad (12)$$

where the values between brackets are our numerical estimation. It should be noted that the difference between the rational exponents and the measured ones are within the range of discrepancy observed when comparing our values of the exactly known exponents corresponding to BD, namely $\beta_{BD} = \frac{1}{3}$ (0.31 ± 0.02), $\alpha_{BD} = \frac{1}{2}$ (0.43 ± 0.05) and $Z_{BD} = \frac{3}{2}$ (1.4 ± 0.1) where the values between brackets are our numerical estimations. Of course, equation (11) implies that only two of the new exponents may be independent.

In order to check our conjectures, figure 4 shows log–log plots of $W(t, L, p)L^{-\alpha_{BD}} p^{\delta}$ versus $t/L^{Z_{BD}} p^{-y}$. The excellent data collapsing obtained for different values of L and p

strongly supports our ansatz given by equation (10). As expected, the failure of data collapsing is observed outside the range of validity of our ansatz, e.g. for $t < t_{x1}$.

It should be noted that the conjectured exact values of the crossover exponents can be obtained using a simple (but misleading) phenomenological argument. In fact, since the RD process does not induce lateral correlations, the relevant time scale for the lateral spreading of correlations should be pt (i.e. the number of BD particles deposited per site) rather than t. It then immediately follows that, at least for small p, the crossover time t_{x1} is of the order of $1/p$, and t_{x2} is of the order of $L^{z_{BD}}/p$. This implies that $y = 1$. Moreover, demanding that the surface width in equation (9) matches the RD behaviour at t_{x1} yields $\gamma = \frac{1}{6}$, and the third exponent follows from the scaling relation (11). However, we would like to stress that this simple argument is misleading because it does not hold for the case of the competitive dynamic between RD and random deposition with relaxation (RDR). In fact, based on extensive numerical simulations we have found [25] that the relevant time scale for the spreading of the correlations in the RD/RDR systems is given by $p^2 t$ with $y_{RD/RDR} = 2$ (instead of pt with $y_{RD/BD} = 1$). So, we conclude that the exponent y is not trivial at all, but it is related to the capacity of the process capable of generating correlations (either BD or RDR) to suppress the fluctuations of the competing RD process [25].

4. Conclusions

Summing up, a competitive growth process is introduced and studied. The properties of the resulting growing interface is rationalized by means of a new phenomenological dynamical scaling approach that involves two new exponents. The proposed ansatz (equation (10)) generalizes the pioneering scaling ansatz of Family and Vicsek [10], and allows us to establish a scaling relation, given by equation (11), between the new exponents and the well known growth exponent of ballistic deposition. We expect that this work will stimulate the study of competitive growth processes and the search of generalized dynamic scaling approaches.

Acknowledgments

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